# Econ 7010 - Midterm 1

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The exam is 70 minutes long, and has 3 questions. Each question is worth the same amount of marks. Upload your answers to the UM Learn dropbox, within 75 minutes of exam start. You may quickly submit a low-quality version of your exam, and then upload a higher quality version after the 75 minutes.

1. Under our various assumptions A.1 to A.6, the sampling distribution of the LS estimator $\boldsymbol{b}$ is:

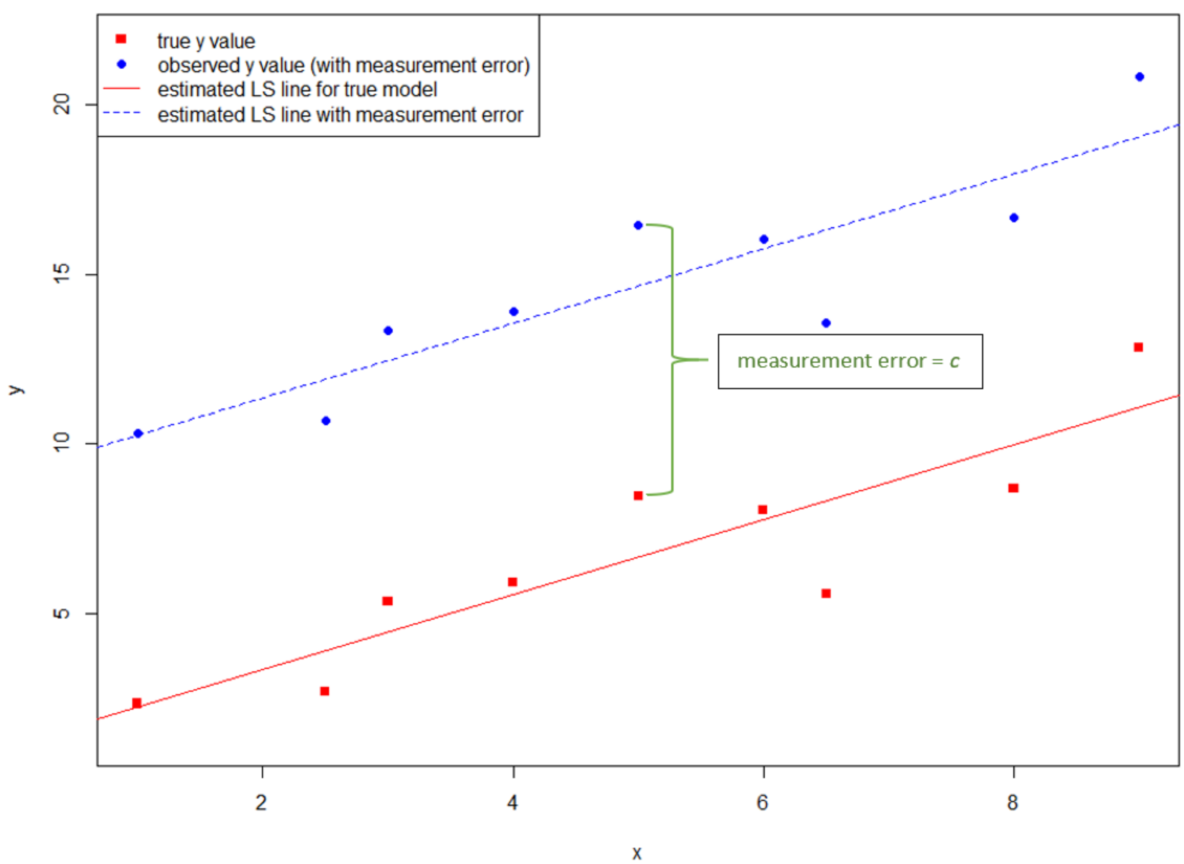
$$\boldsymbol{b} \sim N\left[\boldsymbol{\beta}, \sigma^2(X'X)^{-1}\right]$$

   Prove this result, carefully stating each assumption that you use.

2. Suppose that all of the usual assumptions hold, **except** A.3. That is,

$$E(\boldsymbol{\varepsilon}) = \boldsymbol{i}c,$$

   where $\boldsymbol{i}$ is a column of 1s, and $c$ is a scalar constant. This situation could arise due to *measurement error*, for example. In this case each $y_i$ value is measured to be "too large" by an amount $c$ (or "too small" if $c$ is negative). Below is a picture that may help you visualize the situation:



Now, suppose that you estimate the model:

$$\boldsymbol{y} = \beta_1 + \beta_2 \boldsymbol{x} + \boldsymbol{\varepsilon}$$

a) Is the vector of LS estimators, $\boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, *biased*? Prove your result. If you can't prove, try to explain what you expect to happen intuitively.

b) Are *both* $b_1$ and $b_2$ biased? Prove your result. If you can't prove, try to explain what you expect to happen intuitively.

   Hints:

   - You could rewrite the above model in matrix form as:

   $$\boldsymbol{y} = X_1 \beta_1 + X_2 \beta_2 + \boldsymbol{\varepsilon},$$

   where $X_1$ is just a column of 1s, that is $X_1 = \boldsymbol{i}$. Then, use the partial estimator formulas for $b_1$ and $b_2$.
   - The residuals from a regression of a vector of constants on another vector of constants, are zero. For example, in the model $\boldsymbol{i}c = \boldsymbol{i} + \boldsymbol{\epsilon}$, the LS residuals are all equal to zero.

c) Is the LS estimator still efficient (given that there is measurement error)?

3. The typical LS estimator, $\boldsymbol{b}$, is derived by minimizing the sum of squared *vertical* distances between the estimated line, and each data point ($\boldsymbol{e}'\boldsymbol{e}$).

   a) Explain how you would derive the formula for a different estimator; one which minimizes the sum of squared *horizontal* distances between the estimated line, and each data point. (**Explain how you would derive, and set up the problem, but do not attempt to actually derive the formula for the estimator**.)

   b) What can you say about the variance of this new estimator, compared to the variance of the LS estimator?