Department of Economics **University of Manitoba**

ECON 7010: Econometrics I
Midterm 2, Nov. 5, 2019

Instructor: **Ryan Godwin**
Instructions: **Answer questions in the booklet provided.**
Time Allowed: **75 minutes (Total marks = 98)**
Number of Pages: **2**

PART A: Short answer – **choose 5 out of 7**. [10 marks each]

1.) Carefully explain what it means for a test to be uniformly most powerful.

2.) Given that:

$$E(\hat{\sigma}^2) = \frac{n-k}{n}\sigma^2,$$

prove that s^2 is an unbiased estimator.

3.) Prove that $\bar{y}$ is mean-square consistent (strongly consistent).

4.) What is the benefit to having an instrument matrix, *Z*, that has higher dimension than the *X* matrix? (That is, what is the benefit of having more instruments than endogenous variables?) Explain how you would prove this.

5.) Explain the Hausman-Durbin-Wu test for the necessity of IV. Why shouldn't you use instrumental variable estimation if you don't have to?

6.) Explain how to compare the asymptotic efficiency of two consistent estimators.

7.) Explain why the value for the *F* test statistic cannot be negative.

PART B: Longer Answer – **answer both questions**. [24 marks each]

8.) [Answer 4 out of 5 questions among parts (a) to (e).]

a) Name an assumption required for the F-test statistic to follow the F distribution. If this assumption does not hold, which test can be used instead?

b) See your formula sheet. Derive $E(b_*)$ (the expected value of the RLS estimator). Under what condition is b_* unbiased?

c) Explain why b_* is more precise than the OLS estimator, even if the restrictions are false. Describe how to prove this.

d) Explain why it might be better to adopt a general-to-specific approach to model building (i.e. start with a big model, and eliminate regressors), rather than a specific-to-general approach.

e) Show that the F-statistic may be calculated using the R^2 from a restricted and an unrestricted model.

9.) [Answer 3 out of 4 questions among parts (a) to (d). You must answer part (e).]

Let the population model be:

$$y = X\beta + \varepsilon$$

a) Prove that OLS is inconsistent if X and ε are <u>not</u> independent

b) Suppose that there is available a matrix of instruments, Z. Prove that the IV estimator for β is consistent, carefully stating any assumptions that you use.

c) Derive the simple (just identified) IV estimator using the two-stage-least-squares (2SLS) interpretation.

d) Prove that the generalized (over-identified) IV estimator collapses to the simple one, if X and Z have the same dimension.

e) Suppose that the population model is partitioned so that:

$$y = X_1\beta_1 + X_2\beta_2 + \varepsilon$$

and that:

$$plim\left(\frac{X_1'\varepsilon}{n}\right) = 0,$$

$$plim\left(\frac{X_2'\varepsilon}{n}\right) = Q_{X_2,\varepsilon}\ (\neq 0).$$

Is the OLS estimator for β_1 consistent? Prove. If you can't prove, explain.