Econ 7010 - 2019 Midterm 1 Answer Key

Ryan T. Godwin

1. A, B, or C.
2. B.
3. B.
4. D.
5. B.
6. See section 3.5.2 of the course notes.
7. See section 4.2.1 of the course notes.
8. The Gauss-Markov theorem establishes that the least squares estimator has minimum variance among all linear and unbiased estimators in the linear population model, provided that the standard assumptions (A1 - A6) hold.
9. The LS estimator is:

$$\boldsymbol{b} = \left(X'X\right)^{-1} X'\boldsymbol{y}$$

If the model contains only an intercept, then the X matrix is only a column of 1s. The LS estimator then becomes:

$$\boldsymbol{b} = \left(\begin{bmatrix} 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \frac{1}{n} \sum_{i=1}^{n} y_i = \bar{y}$$

10. See section 4.5.2 of the course notes.
11. There is a part missing in this exam question. It should read:

Let all of the usual assumptions hold. Suppose that the true population model is:

$$\boldsymbol{y} = X_1\boldsymbol{\beta}_1 + X_2\boldsymbol{\beta}_2 + \boldsymbol{\epsilon} \tag{1}$$

but the equation that you actually specify and estimate is:

$$\boldsymbol{y} = X_1\boldsymbol{\beta}_1 + \boldsymbol{u} \tag{2}$$

Show that, in general, $\boldsymbol{b}_1$ is biased.

Answer. The LS estimator for equation (2) is:

$$\begin{aligned} \boldsymbol{b}_1 &= \left(X_1'X_1\right)^{-1} X_1'\boldsymbol{y} = \left(X_1'X_1\right)^{-1} X_1' \left(X_1\boldsymbol{\beta}_1 + X_2\boldsymbol{\beta}_2 + \boldsymbol{\epsilon}\right) \\ &= \boldsymbol{\beta}_1 + \left(X_1'X_1\right)^{-1} X_1'X_2\boldsymbol{\beta}_2 + \left(X_1'X_1\right)^{-1} X_1'\boldsymbol{\epsilon} \\ \mathrm{E}[\boldsymbol{b}_1] &= \boldsymbol{\beta}_1 + \left(X_1'X_1\right)^{-1} X_1'X_2\boldsymbol{\beta}_2 \end{aligned}$$

The LS estimator is in general biased.

12. A regressor(s) has been left out of model (2). The LS estimator for $\boldsymbol{\beta}_1$ is biased unless either (i) X_1 and X_2 are orthogonal, or (ii) $\boldsymbol{\beta}_2 = 0$ (X_2 has no effect on $\boldsymbol{y}$). In either case, the second term in $\mathrm{E}[\boldsymbol{b}_1]$ cancels.

13. This question has a typo. The sampling distribution is:

$$\boldsymbol{b} \sim N\left[\boldsymbol{\beta}, \sigma^2\left(X'X)^{-1}\right)\right]$$

Brief answer. A.1 and A.6 together provide the Normal distribution for the LS estimator, since it is a linear estimator. A.3 and A.5 are used to show that LS is unbiased and has mean $\boldsymbol{\beta}$. A.4 is used to show that the variance covariance matrix of $\boldsymbol{b}$ is $\sigma^2(X'X)^{-1}$.