Econ 7010 - Midterm 1

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The exam is 75 minutes long, with 100 marks total.

Multiple Choice - 2 marks each

1. The Least Squares principle for estimating a regression model, $\boldsymbol{y} = X\boldsymbol{\beta} + \boldsymbol{\epsilon}$, where *all* of the usual assumptions are satisfied:
   (a) Produces unbiased and efficient estimators of $\boldsymbol{\beta}$.
   (b) Involves minimizing the sum of the squares of the LS residuals.
   (c) Produces an estimator for $\boldsymbol{\beta}$ that has a Normal distribution, centered at $\boldsymbol{\beta}$ itself.
   (d) Produces an equal number of positive and negative residuals if the sample size is even.
2. An estimator is:
   (a) a constant parameter from the population.
   (b) random, because the sample is random.
   (c) unbiased and efficient.
   (d) a sampling distribution.
3. Let the X matrix consist of only a column of 1s. Then, $M^0 y$:
   (a) is a vector of 0s.
   (b) is $y - \bar{y}$.
   (c) are the OLS predicted values.
   (d) is undefined.
4. Suppose that we have all of our standard assumptions, and the model includes an intercept. Then
   (a) the OLS residuals sum to zero.
   (b) the fitted regression passes through the sample mean.
   (c) the sample mean of the fitted y values equals the sample mean of the actual y values.
   (d) all of the above.
5. Which of the following will transform a vector, y, into its sum-of-squares?
   (a) yy'
   (b) $y'y$
   (c) $y'My$
   (d) yMy'

Short Answer - 8 marks each

6. Given the population model:

$$\boldsymbol{y} = X\boldsymbol{\beta} + \boldsymbol{\epsilon},$$

derive the OLS estimator for $\boldsymbol{\beta}$. Which assumptions do you need? [Hint: $\frac{\partial(\boldsymbol{a}'\boldsymbol{x})}{\partial \boldsymbol{x}} = \boldsymbol{a}$ and $\frac{\partial(\boldsymbol{x}'A\boldsymbol{x})}{\partial \boldsymbol{x}} = 2A\boldsymbol{x}$.]

7. Prove that the LS residuals sum to zero, if the model includes an intercept.

8. What is the Gauss-Markov theorem?

9. Suppose that we have the population model:

$$\boldsymbol{y} = X\boldsymbol{\beta} + \boldsymbol{\epsilon},$$

but where the X matrix contains only a column of 1s. In this case, prove that $b = \bar{y}$.

10. Using the idea of a constrained minimization problem, explain that R^2 must increase (or stay the same) when a variable is added to the model. Under what circumstance will R^2 stay the same?

11. Let all of the usual assumptions hold. Suppose that the model that you actually specify and estimate by OLS is $\boldsymbol{y} = X_1\boldsymbol{\beta}_1 + \boldsymbol{u}$. Show that, in general, b_1 is biased.

12. Refer to the previous question. Under what circumstance is b_1 unbiased?

Long Answer - 26 marks

13. Under our various assumptions A.1 to A.6, the sampling distribution of the LS estimator $\boldsymbol{b}$ is:

$$\boldsymbol{b} \sim N\left[\boldsymbol{\beta}, \sigma^2 I_n\right]$$

Prove this result, carefully stating each assumption that you use.