Department of Economics **University of Manitoba**

ECON 7010: Econometrics I

FINAL EXAM, Dec. 9th, 2019

Instructor:	**Ryan Godwin**
Instructions:	**Write all answers in the booklet provided.**
Time Allowed:	**3 hours.**
Number of Pages:	**4**

There are a total of 100 marks.

There is some choice in the questions you can answer. Additional questions will not be marked.

PART A: Short answer. ***Answer 5 out of 8 questions.*** *Each question is worth 7 marks.*

1.) Under what circumstances would we use the GLS estimator, and what are the advantages of using GLS as compared with the OLS estimator?

2.) Under what conditions is the restricted-least-squares (RLS) estimator (i) unbiased, and (ii) efficient relative to the OLS estimator?

3.) Suppose that $E[\varepsilon] = c$, where c is a non-zero constant. Prove that the OLS estimator for the intercept is biased, and that the OLS estimators for the slope coefficients are unbiased.

4.) Explain why the sum of squared OLS residuals cannot decrease (and usually increases) if a regressor is removed from the model.

5.) Prove that the simple IV estimator is consistent, carefully explaining the required assumptions.

6.) Explain what a "spurious" regression is, as it relates to the time series topic discussed in class.

7.) Derive the simple IV estimator using the two-stage-least-squares interpretation.

8.) Explain how to implement GLS when the data has been averaged over groups of known size.

PART B: Long answer. **You must answer this question. 20 marks.**

9.) The assumptions underlying the classical linear regression model are:

A.1	Linearity
A.2	Full rank: rank(X) = k
A.3	Errors have zero mean: $E(\varepsilon) = 0$
A.4	Spherical errors
A.5	The process that generates X is unrelated to the process that generates ε
A.6	Normality of errors

Under these assumptions, OLS is a good estimator. Additionally, $s^2(X'X)^{-1}$ is a good estimator for the variance-covariance matrix of the OLS estimator. For each of the assumptions listed above: (i) describe situations where the assumption might be violated, (ii) describe the consequences of the assumption being violated (in terms of estimation, hypothesis testing, etc.), and (iii) describe how the problem might be corrected, or how an alternative estimator might be used to correct the problem (if possible).

Part C: Long Answer. **Answer 3 of the 4 following questions.** 15 marks each. Each part has equal weight unless otherwise stated.

10.) Suppose that $V(\boldsymbol{\varepsilon}) \neq \sigma^2 I$. Instead, heteroscedasticity is present and $V(\boldsymbol{\varepsilon}) = \sigma^2\Omega$.

a) Explain the implications of ignoring heteroskedasticity.

b) Derive $V(\boldsymbol{b})$ for when $V(\boldsymbol{\varepsilon}) = \sigma^2\Omega$.

c) Describe White's test for heteroskedasticity.

d) Suppose that you have an additional variable, z_i, which describes the form of heteroscedasticity,such that:

$$\begin{bmatrix} z_1 & 0 & 0 & \dots & 0 \\ 0 & z_2 & 0 & \dots & 0 \\ 0 & 0 & z_3 & \dots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & & z_n \end{bmatrix}$$

Explain how to transform the model into one that is characteristic of homoscedasticity, and how you could obtain the Generalized Least Squares (GLS) estimates from the transformed model.

e) Prove that the transformed model above satisfies A.4.

11.) Suppose that we have a linear multiple regression model that satisfies all of the usual assumptions, except that the errors follow a first-order autoregressive (AR(1)) process. So:

$$y_t = \beta_1 + \beta_2 x_{2t} + \cdots + \beta_k x_{kt} + \varepsilon_t \qquad (4.1)$$

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t \quad ; \quad u_t \sim i.i.d. N[0\,, \sigma_u^2]$$

a) Explain what implications this error structure has for: (i) the OLS estimator of the coefficient vector; and (ii) the construction of confidence intervals for the coefficients.

b) Derive the variance of ε_t. What condition is required for this variance to be finite?

c) Derive the covariance between ε_t and ε_s, for $t \neq s$.

d) Describe how to implement FGLS for model (4.1).

e) Describe the consequence of including y_{t-1} as a regressor in (4.1), when y_t also follows an AR(1) process.

12.) The probability function for a single y_i value which follows a Normal distribution is:

$$f(y_i \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\tfrac{1}{2\sigma^2}(y_i - \mu)^2\}\,.$$

Consider our usual linear multiple regression model:

$$y = X\beta + \varepsilon \quad ; \quad \varepsilon \sim \mathrm{N}(0,\ \sigma^2 \mathrm{I}).$$

a) Show that the maximum likelihood estimator for β is the same as the OLS estimator for β.

[10 marks]

b) Briefly describe the asymptotic properties of the maximum likelihood estimator for β.

[5 marks]

13.)

a) Suppose that we wish to estimate a regression model by least squares, but that this model is non-linear in the parameters. Explain why we will (generally) need to use a numerical algorithm to obtain the parameter estimates.

b) Using suitable diagrams, derive the Newton-Raphson algorithm.

c) Describe some of the problems that may arise in the application of the Newton-Raphson algorithm.

d) Using an initial value of $\theta_0 = 2$, calculate the first few iterations of the Newton-Raphson algorithm to find the value of θ that minimizes the function:

$$f(\theta) = \theta^3 - 3\theta$$

END.