Department of Economics **University of Manitoba**

ECON 7010: Econometrics I
Midterm 2, Nov. 17, 2016

Instructor: **Ryan Godwin**
Instructions: **Answer questions in the booklet provided.**
Time Allowed: **75 minutes (Total marks = 100)**
Number of Pages: **4**

PART A: Short answer – **choose 5 out of 6**. [10 marks each]

1.) Explain how to implement White's test for heteroskedasticity.

2.) Using an initial value of $\theta_0 = 2$, calculate the first few iterations of the Newton algorithm to find the value of θ that minimizes the function:

$$f(\theta) = \theta^3 - 3\theta.$$

3.) Suppose that $V(\boldsymbol{\varepsilon}) \neq \sigma^2 I$. Explain one way you could implement FGLS without any knowledge of the form of heteroscedasticity.

4.) Consider the simple linear regression model, where all of the usual assumptions are satisfied. Two potential estimators for σ^2 are $s^2 = e'e/(n-k)$ and $\hat{\sigma}^2 = e'e/n$. By taking the expected values of these estimators we find that s^2 is unbiased and $\hat{\sigma}^2$ is biased: $E[s^2] = \sigma^2$ and $E[\hat{\sigma}^2] = \sigma^2\,(n-k)/n$. Assuming that ε is Normally distributed, it can be shown that: $var(s^2) = 2\sigma^4/(n-k)$ and $var(\hat{\sigma}^2) = 2\sigma^4/n$. In the case of Normally distributed errors, are s^2 and $\hat{\sigma}^2$ consistent estimators? Prove.

5.) Describe the properties of the Restricted Least Squares (RLS) estimator, and how you would implement RLS in practice. Why must the sum-of-squared residuals from RLS be greater than or equal to the sum-of-squared residuals from OLS?

6.) Given that

$$\sqrt{n}(\boldsymbol{b} - \boldsymbol{\beta}) \xrightarrow{d} N[\boldsymbol{0}, \sigma^2 Q^{-1}]$$

and

$$plim(s^2) = \sigma^2\,,$$

argue that

$$plim(s^2(X'X)^{-1}) = V[\boldsymbol{b}]\,.$$

PART B: Longer Answer – **choose 2 out of 3**. [25 marks each]

6.) Suppose that we have the standard multiple regression model with all usual assumptions, except that:

$$var(\varepsilon_i) = \sigma^2 x_{i2}$$

or

$$V(\boldsymbol{\varepsilon}) = \sigma^2 diag(X_2).$$

a) In this case, what is the variance of the OLS estimator?

b) What are the consequences of instead assuming that $V(\boldsymbol{\varepsilon}) = \sigma^2 I$?

c) What is the formula for the GLS estimator in this case?

d) One way to write the GLS estimator is $\hat{\beta} = (X_*' X_*)^{-1} X_*' y_*$, where $X_* = PX$ and $y_* = Py$ for a certain $n \times n$ matrix P. Write a mathematical relation involving P and Ω that must hold for $\hat{\beta}$ to be the GLS estimator.

e) What form does P take when $var(\varepsilon_i) = \sigma^2 x_{i2}$?

[5 marks each]

7.) Suppose that we have a linear multiple regression model,

$$y = X\beta + \varepsilon,$$

where the regressors may be random and may be correlated with the errors, even asymptotically. Accordingly, we decide to use a (generalized) I.V. estimator, where Z is the $(n \times L)$ matrix of (possibly random) instruments.

a) Prove, algebraically, that the I.V. estimator is equivalent to the following two-step estimator:

(i) Regress X on Z by OLS, and get the predicted matrix, $\hat{X}$.

(ii) Fit the following artificial model by OLS: $y = \hat{X}\beta + v$.

[10 marks]

b) Prove that the generalized IV estimator collapses to the simple one, if X and Z have the same dimensions.

[10 marks]

c) Briefly describe how you would test to see if IV estimation is needed.

[5 marks]

8.) A regression model with k = 16 independent variables is fit using a panel of seven years of data. The sums of squares for the seven separate regressions and the pooled regression are shown below. The model with the pooled data allows a separate constant for each year.

	1954	1955	1956	1957	1958	1959	1960	All
Observations	65	55	87	95	103	87	78	570
e'e	104	88	206	144	199	308	211	1425

a) Test the hypothesis that the same coefficients apply in every year. Use the table of Chi-square critical values provided at the end of the exam.

[15 marks]

b) Explain how you could perform the hypothesis test in (a) using a single regression.

[10 marks]

Critical values for the Chi-square (χ_J^2) distribution, for various degrees of freedom (J)

Degrees of Freedom	Critical Value	Degrees of Freedom	Critical Value	Degrees of Freedom	Critical Value	Degrees of Freedom	Critical Value
1	3.84	39	54.57	77	98.48	115	141.03
2	5.99	40	55.76	78	99.62	116	142.14
3	7.81	41	56.94	79	100.75	117	143.25
4	9.49	42	58.12	80	101.88	118	144.35
5	11.07	43	59.30	81	103.01	119	145.46
6	12.59	44	60.48	82	104.14	120	146.57
7	14.07	45	61.66	83	105.27	121	147.67
8	15.51	46	62.83	84	106.39	122	148.78
9	16.92	47	64.00	85	107.52	123	149.88
10	18.31	48	65.17	86	108.65	124	150.99
11	19.68	49	66.34	87	109.77	125	152.09
12	21.03	50	67.50	88	110.90	126	153.20
13	22.36	51	68.67	89	112.02	127	154.30
14	23.68	52	69.83	90	113.15	128	155.40
15	25.00	53	70.99	91	114.27	129	156.51
16	26.30	54	72.15	92	115.39	130	157.61
17	27.59	55	73.31	93	116.51	131	158.71
18	28.87	56	74.47	94	117.63	132	159.81
19	30.14	57	75.62	95	118.75	133	160.91
20	31.41	58	76.78	96	119.87	134	162.02
21	32.67	59	77.93	97	120.99	135	163.12
22	33.92	60	79.08	98	122.11	136	164.22
23	35.17	61	80.23	99	123.23	137	165.32
24	36.42	62	81.38	100	124.34	138	166.42
25	37.65	63	82.53	101	125.46	139	167.51
26	38.89	64	83.68	102	126.57	140	168.61
27	40.11	65	84.82	103	127.69	141	169.71
28	41.34	66	85.96	104	128.80	142	170.81
29	42.56	67	87.11	105	129.92	143	171.91
30	43.77	68	88.25	106	131.03	144	173.00
31	44.99	69	89.39	107	132.14	145	174.10
32	46.19	70	90.53	108	133.26	146	175.20
33	47.40	71	91.67	109	134.37	147	176.29
34	48.60	72	92.81	110	135.48	148	177.39
35	49.80	73	93.95	111	136.59	149	178.49
36	51.00	74	95.08	112	137.70	150	179.58
37	52.19	75	96.22	113	138.81	151	180.68
38	53.38	76	97.35	114	139.92	152	181.77