Econ 7010 - Final Exam 2015 - Answer Key

Part A: Short Answer

1.) The basic idea behind the simple bootstrap is to use the sample of observations as if it were the entire population, so that repeatedly drawing re-samples (with replacement) emulates repeated sampling from the population. ~~In this way~~ By drawing many bootstrap re-samples from the original data, and by calculating the statistic of interest each time, an empirical sampling distribution for the statistic can be observed. This "bootstrap distribution" may be used to bias-correct, calculate p-values, construct confidence intervals, etc.

To construct a confidence interval, for example, you would sort all of the bootstrap statistics in ascending order, and determine the 2.5 and 97.5 percentiles (for a 95% C.I.).

2.) OLS doesn't work very well because it ignores the discrete nature of the data, and the non-negativity of the data. This leads to inefficiency. Not only is OLS inefficient, but the fitted model does not provide the sort of predictions we are looking for.

3.) $R_u^2 = 1 - \dfrac{e'e}{y'M_0y} \Rightarrow e'e = (1 - R_u^2)\, y'M_0y$

$R_R^2 = 1 - \dfrac{e_*'e_*}{y'M_0y} \Rightarrow e_*'e_* = (1 - R_R^2)\, y'M_0y$

$$F = \frac{(1-R_R^2)\cancel{y'M_0y} - (1-R_u^2)\cancel{y'M_0y}}{(1-R_u^2)\cancel{y'M_0y}} \cdot \frac{(n-k)}{J}$$

$$= \frac{(R_u^2 - R_R^2)/J}{(1-R_u^2)/(n-k)}$$

Part B – Long Answer

1.) a) We can partition the X data as follows:

$$X = [X \vdots D], \text{ where } D = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

The OLS estimator is then partitioned as $b = \begin{bmatrix} b_1 \\ \cdots \\ b_2 \end{bmatrix}$.

The OLS estimator associated with "X" is:

$$b_1 = (X' M_D X)^{-1} X' M_D y,$$

where $M_D = I - D(D'D)^{-1}D'$.

Now, $(D'D)^{-1} = 1$ (scalar), and

$$DD' = \begin{bmatrix} 0_{0} & & 0 \\ & \ddots & \\ 0 & & 0_{1} \end{bmatrix}, \text{ so}$$

$$M_D = \begin{bmatrix} 1_{1} & & 0 \\ & \ddots & \\ 0 & & 1_{0} \end{bmatrix}.$$

Using the idempotent property of M_D, we can write

$$b_1 = (X' M_D' M_D X)^{-1} X' M_D' M_D y = (X_*' X_*)^{-1} X_*' y_*,$$

where $X_* = M_D X$ and $y_* = M_D y$.

The transformed data (y^*, X^*) is just the original data, excluding the last observation.

b) $X'e = 0$, always. Since X contains D:

$D'e = 0$, and $e_n = 0$.

3.) Every AR(1) process can be written as an MA(∞) process:

a)

$$\varepsilon_t = \rho \varepsilon_{t-1} + u_t$$

$$\varepsilon_{t-1} = \rho \varepsilon_{t-2} + u_{t-1}$$

$$\begin{aligned} \varepsilon_t &= u_t + \rho u_{t-1} + \rho^2 \varepsilon_{t-2} \\ &= u_t + \rho u_{t-1} + \rho^2 u_{t-2} + \rho^3 \varepsilon_{t-2} \\ &= u_t + \rho u_{t-1} + \rho^2 u_{t-2} + \dots \end{aligned}$$

Writing the AR(1) process in this way shows that ε_t embodies the entire history of the u_t's.

b) We will take the variance of an MA(∞) process, since AR(1) = MA(∞):

$$var(\varepsilon_t) = var(u_t + \rho u_{t-1} + \rho^2 u_{t-2} + \dots).$$

Since u_t is i.i.d.: $var(\varepsilon_t) = var(u_t) + var(\rho u_{t-1}) + var(\rho^2 u_{t-2}) + \dots$

$$var(\varepsilon_t) = var(u_t) + \rho^2 var(u_{t-1}) + \rho^4 var(u_{t-2}).$$

Since $var(u_t) = \sigma_u^2 \ \forall t$,

$$var(\varepsilon_t) = \sigma_u^2 (1 + \rho^2 + \rho^4 + \dots) = \frac{\sigma_u^2}{1-\rho^2}$$

In order for $var(\varepsilon_t)$ to be finite, $|\rho| \neq 1$.

c) In the presence of autocorrelation, OLS is still unbiased and consistent. Hence, the OLS residuals, e, are consistent estimators for the unobservable error term, ε. An observable model for the AR(1) process is:

$$e_t = \rho e_{t-1} + v_t$$

The OLS estimator for ρ is:

$$\hat{\rho} = \frac{\Sigma e_t e_{t-1}}{\Sigma (e_{t-1}^2)}$$

d) $$\varepsilon^* = \begin{bmatrix} \varepsilon_1\sqrt{1-\rho^2} \\ \varepsilon_2 - \rho\varepsilon_1 \\ \varepsilon_3 - \rho\varepsilon_2 \\ \vdots \end{bmatrix}$$

$$\text{cov}(\varepsilon_3^*, \varepsilon_2^*) \text{ (for example) } = E(\varepsilon_3^* \varepsilon_2^*)$$

$$= E[(\varepsilon_3 - \rho\varepsilon_2)(\varepsilon_2 - \rho\varepsilon_1)]$$

Now, $\varepsilon_3 = \rho\varepsilon_2 + u_3$ and $\rho\varepsilon_1 = \varepsilon_2 - u_2$, so:

$$\text{cov}(\varepsilon_3^*, \varepsilon_2^*) = E[(\rho\varepsilon_2 + u_3 - \rho\varepsilon_2)(\varepsilon_2 - \varepsilon_2 + u_2)]$$

$$= E[u_3 u_2] = \text{cov}(u_3, u_2) = 0 \quad (\text{since } u_t \text{ is i.i.d.})$$

4.) a) In the presence of heteroskedasticity OLS is inefficient, however, it is still unbiased and consistent.

Estimation of the variance-covariance matrix of the OLS estimator is inconsistent, if homoskedasticity is assumed. This is because, under het., $var(b) \neq \sigma^2 (X'X)^{-1}$. Basing inference from this formula leads to invalid hypothesis testing.

b) White's Heteroskedasticity Test:

$H_0: \sigma_i^2 = \sigma^2$ $\quad H_A:$ not H_0

It is an asymptotically valid, "non-constructive" test.

To implement the test:

1. Estimate by OLS, get residuals e_i
2. Using OLS, regress e_i^2 on each x, their squared values, and their cross-products
3. nR^2 from step 2 is χ^2_P.

c) Goldfeld-Quandt Test:

Used when the sample is potentially from two different populations, where the populations differ only in σ^2 (not β).

$H_0: \sigma_1^2 = \sigma_2^2$ $\quad$ $H_A: \sigma_1^2 \neq \sigma_2^2$

It is a "constructive" test.

To implement:

1. Fit the model by OLS over the two samples, get e_1 and e_2
2. Construct the statistic:

$$F = \frac{s_1^2}{s_2^2}$$

d) We could either multiply all data in the first sample by $\frac{s_2}{s_1}$, or data in 2^{nd} sample by $\frac{s_1}{s_2}$, and apply OLS to the transformed data.

5. See answer key for assignment 3.