

Department of Economics

University of Manitoba

ECON 7010: Econometrics I
Midterm, Oct. 06, 2014

Instructor: Ryan Godwin
Instructions: Answer ALL QUESTIONS, and put all answers in the booklet provided
Time Allowed: 75 minutes (Total marks = 72 – i.e., ~ one mark per minute)
Number of Pages: 3

PART A:

Select the most appropriate answer in each case. Each question is worth 3 marks. (No explanation is needed to obtain full marks, but it will be taken into account if given.)

1.) If the regression model includes an intercept, then:

- a) The OLS residuals sum to zero.
- b) The fitted regression passes through the sample mean.
- c) The sample mean of the fitted y-values equals the sample mean of the actual y-values.
- d) All of the above.

2.) The formula for the OLS estimator: $\mathbf{b} = (X'X)^{-1}X'\mathbf{y}$, is derived by:

- a) Ensuring that \mathbf{b} is a linear, unbiased and efficient estimator for $\boldsymbol{\beta}$.
- b) Minimizing the sum of squared residuals.
- c) Minimizing bias and variance.
- d) Ensuring that \mathbf{b} has the best fit (which also maximizes R^2).

3.) The probability of a “Type I” for a t-test:

- a) Is one minus the power of the test, when the null hypothesis is false.
- b) Is equal to the p-value of the test, when the null hypothesis is true.
- c) Is not higher than the power of the test.
- d) Is determined by the standard error of the regression.

4.) A p-value is:

- a) The probability of calculating a test statistic more extreme than the one just calculated.
- b) The maximum and minimum values for the test statistic, that won't be rejected in a hypothesis test.
- c) The maximum and minimum values for the null hypothesis, that won't be rejected in a hypothesis test.
- d) Equal to the probability of a type I error.

PART B: Answer all questions.

5.) Consider the population model:

$$\mathbf{y} = X\boldsymbol{\beta} + \boldsymbol{\varepsilon},$$

where \mathbf{b} is the OLS estimator for $\boldsymbol{\beta}$.

- a) Show that the variance-covariance matrix of \mathbf{b} is equal to $\sigma^2(X'X)^{-1}$. Which assumptions have you used?

[10 marks]

- b) Let $\tilde{\boldsymbol{\beta}}$ be any other estimator for $\boldsymbol{\beta}$ (not the OLS estimator). What can you say about the variance of $\tilde{\boldsymbol{\beta}}$?

[5 marks]

6.)

- a) Explain why R^2 cannot decrease when a regressor is added to the regression model.

[5 marks]

- b) Suppose that for an OLS regression, the $R^2 = 1$. What must the vector of residuals, \mathbf{e} , be equal to?

[5 marks]

7.) Let $\hat{\theta}$ be an estimator for the population parameter θ . Suppose that:

$$E(\hat{\theta}) = \frac{n-c}{n}\theta,$$

where n is the sample size, and c is a constant. Using $\hat{\theta}$, construct an unbiased estimator for θ .

[5 marks]

8.) Let $M = I - X(X'X)^{-1}X'$. Show that $MX = 0$ (you may either prove it algebraically, or explain it intuitively).

[10 marks]

9.) Let \mathbf{e} be the residual vector associated with the ordinary least squares estimation of the linear multiple regression model, $\mathbf{y} = X\boldsymbol{\beta} + \boldsymbol{\varepsilon}$.

(a) Prove that $E(\mathbf{e}) = \mathbf{0}$, and carefully state which of our usual assumptions you are making.

[10 marks]

(b) Prove that $X'\mathbf{e} = \mathbf{0}$, and carefully state which of our usual assumptions you are making.

[10 marks]

END.