**Department of Economics** **University of Manitoba**

**ECON 7010: Econometrics I**
**FINAL EXAM, Dec. 6th, 2013**

**Instructor:** **Ryan Godwin**
**Instructions:** **Put all answers in the booklet provided.**
**Time Allowed:** **3 hours.**
**Number of Pages:** **5**

**There are a total of 100 marks.**

---

**PART A:** *Short answer.* ***Answer 6 out of 10 questions.*** *Each question is worth 6 marks.*

**1.)** Explain whether the following statement is true or false:

"Testing the significance of the coefficient of a zero-one dummy variable that is included in the model to account for a possible structural break at a single known point is equivalent to applying the Chow test for structural stability."

**2.)** Explain what we mean by "nested models" (or "nested hypotheses"). Explain why the sum of squared residuals for a restricted model must higher than the sum of squared residuals for an unrestricted model, when the models are nested.

**3.)** Suppose that all of the usual assumptions underlying OLS are satisfied, except that:

$$V(\varepsilon) = \sigma^2\Omega.$$

Suppose that we had reason to believe that

$$\Omega = \begin{pmatrix} \sigma_1^2 I_{n_1} & 0 & 0 \\ 0 & \sigma_2^2 I_{n_2} & 0 \\ 0 & 0 & \sigma_3^2 I_{n_3} \end{pmatrix} \quad ; \quad n_1 + n_2 + n_3 = n$$

Explain in detail how we would implement a feasible GLS estimator for $\beta$.

**4.)** Suppose that we have a linear regression model, with $k$ regressors:

$$y = X\beta + \varepsilon,$$

and we are concerned with $J$ linear restrictions on $\beta$, of the form $R\beta = q$. Let $b$ be the OLS estimator of $\beta$, and let $b^*$ be the corresponding Restricted Least Squares (RLS) estimator. (*See your formulae sheet for the formula for the RLS estimator*.) Under what conditions is $b^*$ an unbiased estimator of $\beta$?

**5.)** Discuss some of the consequences of estimating a least squares regression model when (i) we wrongly force the regression "line" to pass through the origin; and (ii) we allow for a non-zero intercept, when really it should be zero.

**6.)** Suppose that the marginal effect of $X$ on $y$ is non-linear. For example:

$$\frac{\partial y}{\partial X} = \beta_1 + \beta_2 X$$

Do you necessarily need to estimate a non-linear model in order to capture this non-linear marginal effect? Explain why or why not.

**7.)** Suppose that we wish to estimate a regression model by least squares, but this model is non-linear in the parameters. Explain why we will (generally) need to use a numerical algorithm to obtain the parameter estimates. Using appropriate graphs, explain how Newton's algorithm works.

**8.)** Explain how you would perform White's test for heteroskedasticity.

**9.)** Prove that the OLS estimator for $\beta$ is consistent, stating any assumptions you use.

**10.)** Derive the I.V. estimator for $\beta$, in the case of over-identification (i.e. when there are more instruments than endogenous variables).

**Choose 6 questions from above. For PART A, I will only mark the first 6 questions in your exam booklet.**

**PART B. ANSWER ALL QUESTIONS.**

**11.)** The assumptions underlying the classical linear regression model are:

A.1 Linearity
A.2 Full rank: rank($X$) = $k$
A.3 Errors have zero mean: $E(\varepsilon) = 0$
A.4 Spherical errors
A.5 The process that generates $X$ is unrelated to the process that generates $\varepsilon$
A.6 Normality of errors

Under these assumptions, estimation of the linear model by OLS is sensible. Estimation of the variance of ***b*** by $s^2(X'X)^{-1}$ is also sensible. For each of the assumptions listed above, explain: (i) how the assumption might be violated, (ii) the implications for estimating the model by OLS, and (iii) how the problem might be corrected, or how an alternative estimator might be used to correct the problem (if possible).

20 marks

**12.)** The probability density function for a random variable, $y_i$, is given by:

$$p(y_i|\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(y_i-\mu)^2}{2\sigma^2}},$$

with $E[y_i] = \mu$ and $var[y_i] = \sigma^2$. Suppose that the mean of $y$ is known to be zero. That is, $E[y_i] = \mu = 0$. In this case, the probability density function becomes:

$$p(y_i|\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-y_i^2}{2\sigma^2}}.$$

a) Solve for the maximum likelihood estimator of $\sigma$ (or $\sigma^2$), in the case where $\mu = 0$.

5 marks

b) Do you think the maximum likelihood estimator is biased in this situation?

2 marks

c) Is there an easy way to find the maximum likelihood estimator for $\sigma^4$? Briefly explain.

2 marks

d) Explain the intuition behind the Langrange Multiplier test.

4 marks

**13.)** Suppose that the true data-generating process is

$$y = X_1\beta_1 + X_2\beta_2 + \varepsilon \quad ; \quad \varepsilon \sim N(0, \sigma^2 I), \tag{1}$$

where all of the regressors are non-random, and both $X_1$ and $X_2$ have full rank.

However, the model that we estimate by mistake, using *Generalized Least Squares,* and **assuming** that $V(\varepsilon) = \sigma^2\Omega$, is

$$y = X_1\beta_1 + \nu. \tag{2}$$

(a) Derive the expression for the covariance matrix of this GLS estimator of $\beta_1$. (Hint: note that under model (2), we are assuming the wrong covariance matrix.)

4 marks

(b) Prove that this GLS estimator is biased, in general. Under what conditions will it be unbiased?

3 marks

(c) Derive the expression for the covariance matrix of the estimator of $\beta_1$ obtained if we had applied OLS to (1). (Hint: use $M_2$ when writing the equation for $b_1$.)

4 marks

(d) Now comparing the GLS estimator of $\beta_1$ in (2), and the OLS estimator of $\beta_1$ in (1), what can you say about their relative variances? (No formal proofs are needed.)

4 marks

**14.)**

The following EViews results relate to a model that explains the net worth of a cross-section of U.S. individuals in 1989, measured in thousands of dollars. The regressors are:

EDUC = years of education
MARRIED = dummy variable (= 1 if married; = 0 if not)
PYEARS = number of years in a pension plan
AGE = age, in years
AFRAM = dummy variable (=1 if African American; = 0 if not)
PCTSTCK = % of pension plan held in stocks
FINC101 = dummy variable (= 1 if family income > $100,000 p.a.; = 0, otherwise)

Dependent Variable: NET_WORTH
Method: Least Squares
Date: 10/22/08 Time: 10:05
Sample: 1 226
Included observations: 205

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | -679.1467 | 259.2987 | -2.619167 | 0.0095 |
| EDUC | 17.47498 | 6.184717 | 2.825511 | 0.0052 |
| MARRIED | 99.17807 | | | 0.0095 |
| PYEARS | -3.350712 | 1.675122 | -2.000280 | 0.0468 |
| AGE | 10.39204 | 3.982739 | 2.609270 | 0.0098 |
| AFRAM | -96.92262 | 51.90103 | -1.867451 | 0.0633 |
| PCTSTCK | -0.198055 | 0.398787 | -0.496644 | 0.6200 |
| FINC101 | 180.2197 | 70.46655 | 2.557521 | 0.0113 |

| | | | |
|---|---|---|---|
| R-squared | | Mean dependent var | 210.8534 |
| Adjusted R-squared | 0.158546 | S.D. dependent var | 242.1072 |
| S.E. of regression | 222.0869 | Akaike info criterion | 13.68226 |
| Sum squared resid | 9716547. | Schwarz criterion | 13.81194 |
| Log likelihood | -1394.431 | Hannan-Quinn criter. | 13.73471 |
| F-statistic | 6.491077 | Durbin-Watson stat | 1.952117 |
| Prob(F-statistic) | 0.000001 | | |

a) Calculate the value for the "missing" $R^2$, and explain what this value tells us.

4 marks

b) Construct a 95% confidence interval for the coefficient of AGE, and carefully interpret its meaning.

4 marks

c) Test the hypothesis that the coefficient of EDUC equals 10, using a 2-sided alternative hypothesis, and a 5% significance level.

4 marks

d) Determine the *p*-value for a test of the null hypothesis that the coefficients on EDUC, MARRIED, PYEARS, AGE, AFRAM, PCTSTCK, and FINC101, are jointly equal to zero.

4 marks

**END**