

Econ 3040 A01 - Midterm - Fall 2022

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The exam is 70 minutes long, and consists of 72 marks (**approximately 1 mark per minute**). There are 9 short answer questions, each worth 4 marks. There is one long answer question with 9 parts, each part worth 4 marks. Write all answers in the provided exam booklet. You may only have a calculator and writing implements at your table. You may not use any books, notes, formula sheets, computers, or phones. A table of areas under the standard Normal curve is provided at the back of the exam.

DO NOT OPEN THIS EXAM BOOKLET UNTIL INSTRUCTED TO DO SO.

DON'T TOUCH! (Until instructed to do so).

Short Answer

1. A random variable Y is equal to 2 with probability 0.25, equal to 4 with probability 0.75. What is the expected value, and variance, of Y ?
2. X and Y are two random variables, and their joint probability function is:

	$Y = -50$	$Y = 2$
$X = -4$	0.70	0
$X = 10$	0	0.30

What is the correlation between X and Y ? Explain.

3. Explain why estimators are random variables.
4. How are the formulas for the least-squares estimators derived?
5. What is the role of ϵ in the population model?
6. What is a least squares “predicted value”, and what is a “residual”?
7. What are some factors that might effect the precision (variability) of the least-squares slope estimator?
8. Why is the least-squares estimator “good”? That is, why should we use the least-squares estimator instead of some other estimator?
9. Describe a situation where R^2 would be equal to 1.

Long Answer

10. This question uses a dataset with $n = 2000$ and three variables: **wage** - the hourly wage of a worker in dollars, **education** - the number of years of education of the worker, and **economics** - a dummy variable taking on the value 1 if the worker has an economics degree, and 0 otherwise.

A least-squares model is estimated using R:

```
summary(lm(wage ~ education), data = mydata)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.35165	0.98614	0.357	0.721
education	2.25278	0.06464	34.849	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9.048 on 1998 degrees of freedom
Multiple R-squared: 0.378, Adjusted R-squared: 0.3777
F-statistic: 1214 on 1 and 1998 DF, p-value: < 2.2e-16

Use the above R code and output to answer questions (a) to (e).

- What population model has been estimated?
- What is the interpretation of the number 2.25278?
- How much of the variation in wage can be explained by years of education?
- Test the hypothesis that education has no effect on wage.
- Use the above estimated model to predict how much a worker with 16 years of education will make.

Now, a model using the dummy variable is estimated (use this information to answer questions (f) to (i)):

```
summary(lm(wage ~ economics), data = mydata)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	33.6273	0.2690	124.986	< 2e-16 ***
economics	3.6306	0.8551	4.246	2.28e-05 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11.42 on 1998 degrees of freedom
Multiple R-squared: 0.008942, Adjusted R-squared: 0.008446
F-statistic: 18.03 on 1 and 1998 DF, p-value: 2.277e-05

- Is the dummy variable **economics** statistically significant?
- What is the sample average wage for workers **without** an economics degree? What is the sample average wage for workers **with** an economics degree?
- An economics department is claiming that it's economics graduates make \$4 per hour more than other graduates. Test this hypothesis using the above output.
- Construct a 95% confidence interval around b_1 .

END

Table 1: Area under the standard normal curve, to the right of z .

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002