Econ 3040 A01 - Midterm

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The exam is 65 minutes long, and consists of 65 marks (**1 mark per minute**). Upload your answers to the UM Learn dropbox, within 75 minutes of exam start. You may quickly submit a low-quality version of your exam, and then upload a higher quality version after the 75 minutes, as long as there are no differences between the versions. **Do not share this midterm with anyone.**

1. Players of Dungeons and Dragons use dice that have more than just 6 sides. Consider an 8-sided die that has the values $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

    a) [**2 marks**] Find the expected value of an 8-sided die roll.

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    The variance of an 8-sided die roll is 5.25.

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    b) [**3 marks**] Write down the probability function for an 8-sided die roll. What two important things does a probability function accomplish?

    c) [**3 marks**] What is the expected value and variance of the **sum** of two 8-sided dice?

    d) [**3 marks**] When you roll two 8-sided dice, what is the correlation between the result of the first roll and the second roll? Write one sentence that carefully explains how you know what this correlation is.

    e) [**3 marks**] Imagine that you roll 200 8-sided dice all at once, and sum the results. What probability distribution will this sum follow, and why?

2. X and Y are two random variables, and their joint probability function is:

	$Y = -2$	$Y = 0$	$Y = 2$
$X = 2$	0.25	0	0.25
$X = -2$	0	0.5	0

    The covariance between X and Y is zero.

    a) [**3 marks**] Are X and Y independent? Why or why not?

    b) [**2 marks**] Does X *cause* Y? Explain in one sentence.

3. [**4 marks**] The estimators $\bar{y}$, b_0, and b_1 are random, and have *sampling distributions* (a special name for a probability function). Why is it so important to understand that estimators are random variables? (That is, what kind of results can we derive only by realizing that estimators are random variables?)

4. The University of Mars claims that its students receive, on average, a scholarship of \$300. You collect a sample of scholarship amounts from 200 randomly selected students. You calculate the sample average scholarship to be \$293.5, and sample variance to be \$111063.1.

a) [**3 marks**] The number \$111063.1 was calculated using a formula with $(n-1)$ in the denominator. Why is the denominator $(n-1)$ and not, for example, n?

b) [**6 marks**] Test the null hypothesis that the population mean scholarship amount is \$300. Calculate the t-test statistic, and associated p-value. Decide whether to reject or fail to reject the null hypothesis.

c) [**3 marks**] Calculate the 95% confidence interval around the sample mean. Without doing any additional work, decide whether to reject or fail to reject the null hypothesis that the population mean scholarship is \$240, at the 5% significance level.

d) [**3 marks**] Hackers obtain the entire database of scholarship amounts for all 31,254 University of Mars students. The true population mean scholarship amount is actually \$290.93! Carefully explain, using one or two sentences, the result of your hypothesis test in light of the true population mean.

5. Use the following data: $Y = \{-1, -3, 3\}$; $X = \{-2, 0, 1\}$.

   a) [**3 marks**] Calculate the OLS estimates b_0 and b_1.
   b) [**2 marks**] Calculate the OLS predicted value, and residual, for the 3rd observation.
   c) [**3 marks**] Where did the formulas for b_0 and b_1 come from?

6. [**4 marks**] Use the following **incomplete** data for this question: $Y = \{2, 3, ?\}$; $X = \{1, 2, ?\}$. The OLS residuals associated with this data are: $e = \{0, 0, 0\}$. What are b_0 and b_1?

7. You have been hired by the government to determine how much less alcohol per capita will be consumed if taxes are increased by \$2. You collect data on the quantity consumed (Q, measured in litres per year) at different price levels (P).

   a) [**3 marks**] Write down the population model that you need to estimate.
   b) [**3 marks**] Why should you use OLS to estimate this model?
   c) [**3 marks**] What are some factors that might be in ϵ?

   The estimated model is $\hat{Q} = 15.28 - 0.75P$.

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   d) [**3 marks**] What do you tell the government?
   e) [**3 marks**] How much alcohol will be consumed if the price is set at \$10?

END