6 – Multiple Regression

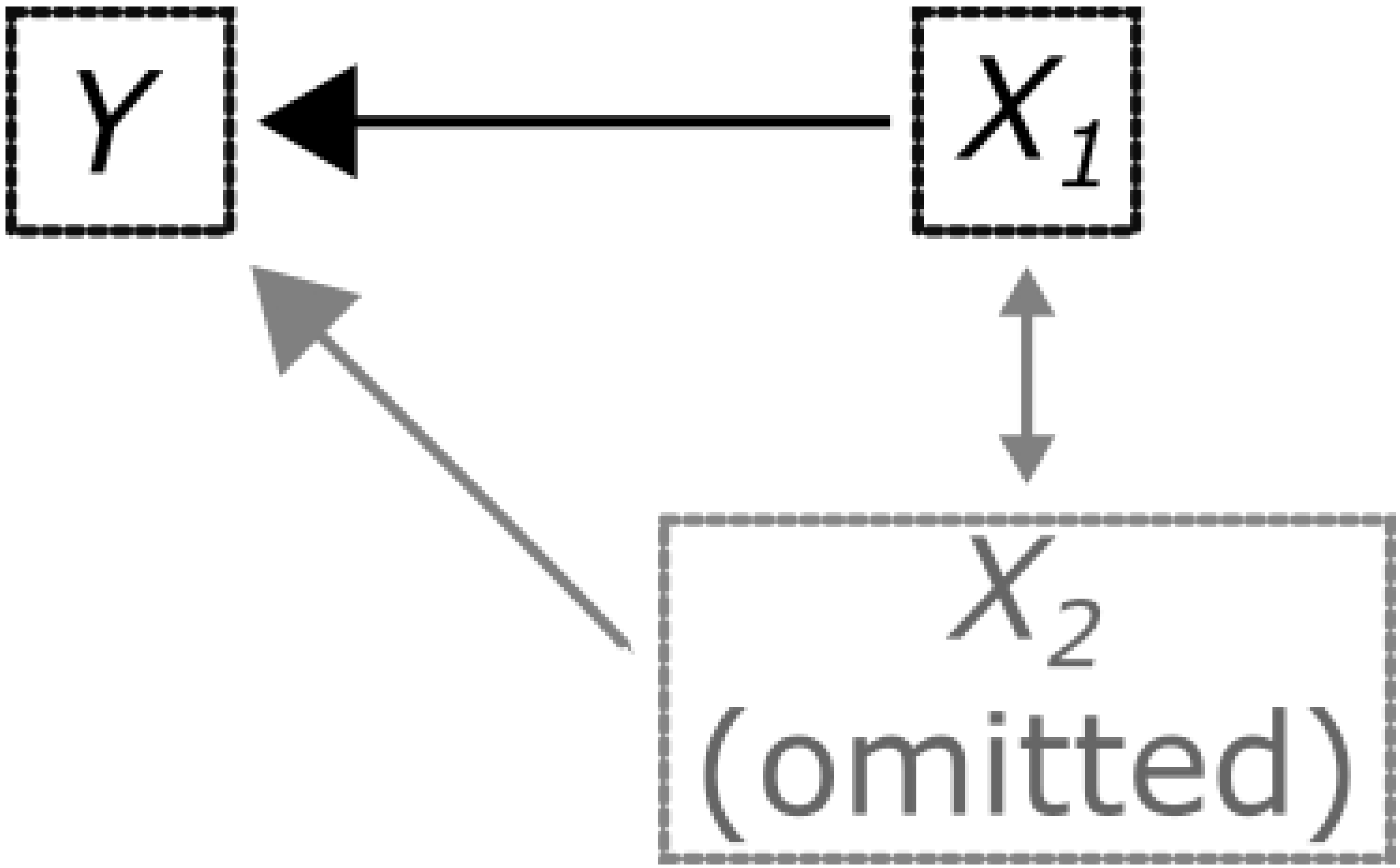
More than one “X” variable.

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_k X_{ki} + \epsilon_i \tag{6.1}$$

Why?

- Might be interested in more than one marginal effect
- Omitted Variable Bias (OVB)

An omitted X_2 variable that is correlated with X_1, and that also determines Y, will make estimation of the true effect of X_1 on Y impossible.



6.1 and 6.2 – House prices and OVB

Should I build a fireplace?

The following empirical example uses data on house prices, in the New York area in 2002-2003 (the data are from Richard De Veaux of Williams College).

Let’s try to determine the value of a fireplace. First, load the data and take a look at it.

```
houses <-
read.csv("http://rtgodwin.com/data/houseprice.csv")
```

```
head(houses)
```

The “head” command prints out the first 6 observations from each variable. You should see something like:

Price	Lot.Size	Waterfront	Age	Land.Value	New.Construct
132500	0.09	0	42	50000	0
181115	0.92	0	0	22300	0
109000	0.19	0	133	7300	0
155000	0.41	0	13	18700	0
86060	0.11	0	0	15000	1
120000	0.68	0	31	14000	0

Central.Air	Fuel.Type	Heat.Type	Sewer.Type	Living.Area	Pct.College
0	3	4	2	906	35
0	2	3	2	1953	51
0	2	3	3	1944	51
0	2	2	2	1944	51
1	2	2	3	840	51
0	2	2	2	1152	22

Bedrooms	Fireplaces	Bathrooms	Rooms
2	1	1.0	5
3	0	2.5	6
4	1	1.0	8
3	1	1.0	5
2	0	1.0	3
4	1	1.0	8

We are interested in the effect of the variable *Fireplaces* on *Price*.

Is *Fireplaces* a dummy variable?

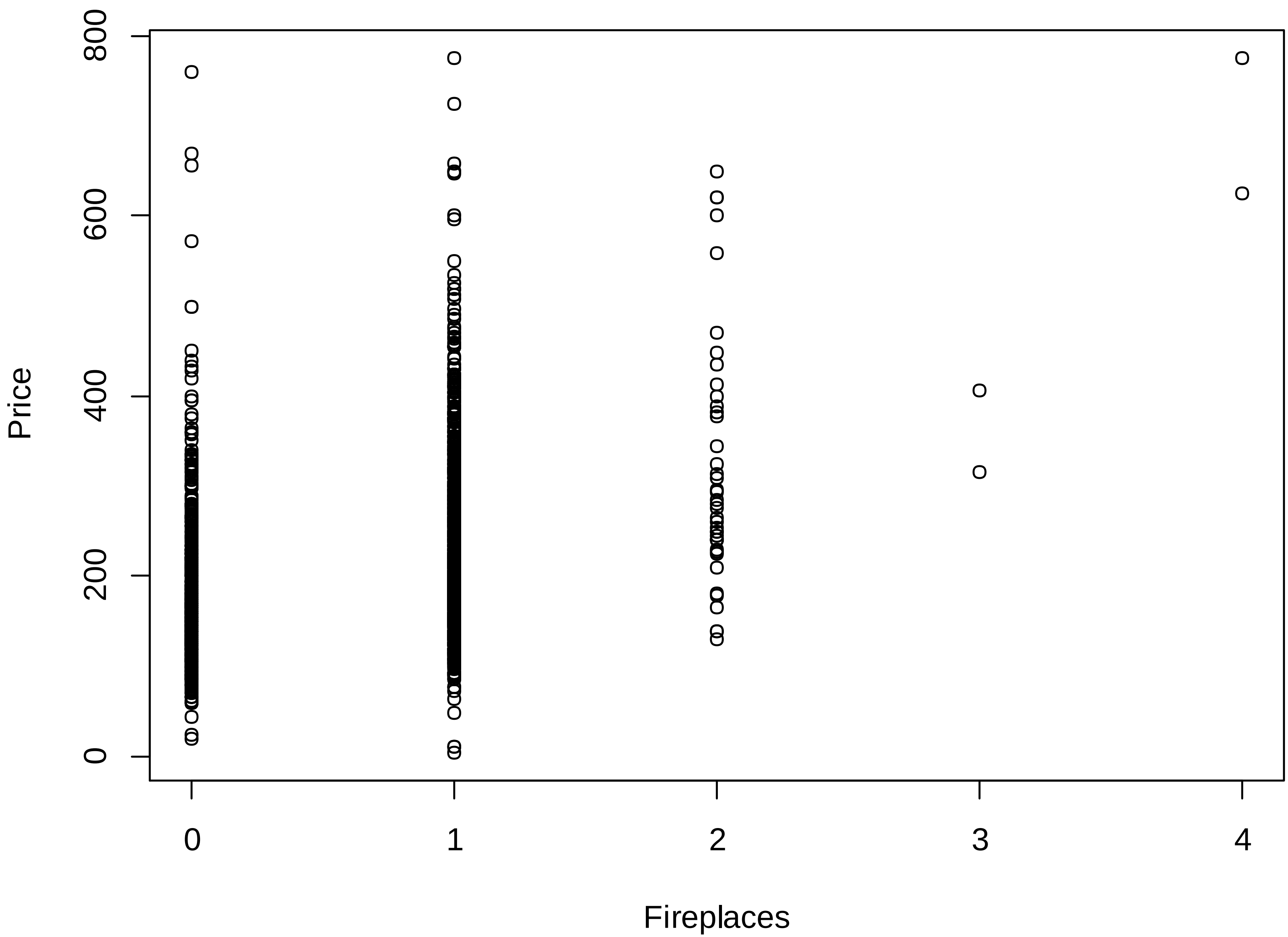
```
summary(house$Fireplaces)
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.0000  0.0000  1.0000  0.6019  1.0000  4.0000
```

Before we proceed, let's instead measure *Price* in thousands of dollars:

```
house$Price = house$Price / 1000
```

Now, let's see the relationship between *Fireplaces* and *Price*.

```
plot(house$Fireplaces, house$Price)
```


Price
Fireplaces
0
200
400
600
800
0
1
2
3
4

Let's see the average Price conditional on different number of Fireplaces:

```
mean(house$Price[house$Fireplaces == 0])
[1] 174.6533
mean(house$Price[house$Fireplaces == 1])
[1] 235.1629
mean(house$Price[house$Fireplaces == 2])
[1] 318.8214
mean(house$Price[house$Fireplaces == 3])
[1] 360.5
mean(house$Price[house$Fireplaces == 4])
[1] 700
```

Correlation?

```
cor(house$Price, house$Fireplaces)
[1] 0.3767862
```

It appears that the more Fireplaces, the higher the Price.

Let's try estimating the population model:

$$Price_i = \beta_0 + \beta_1 Fireplaces_i + \epsilon_i$$

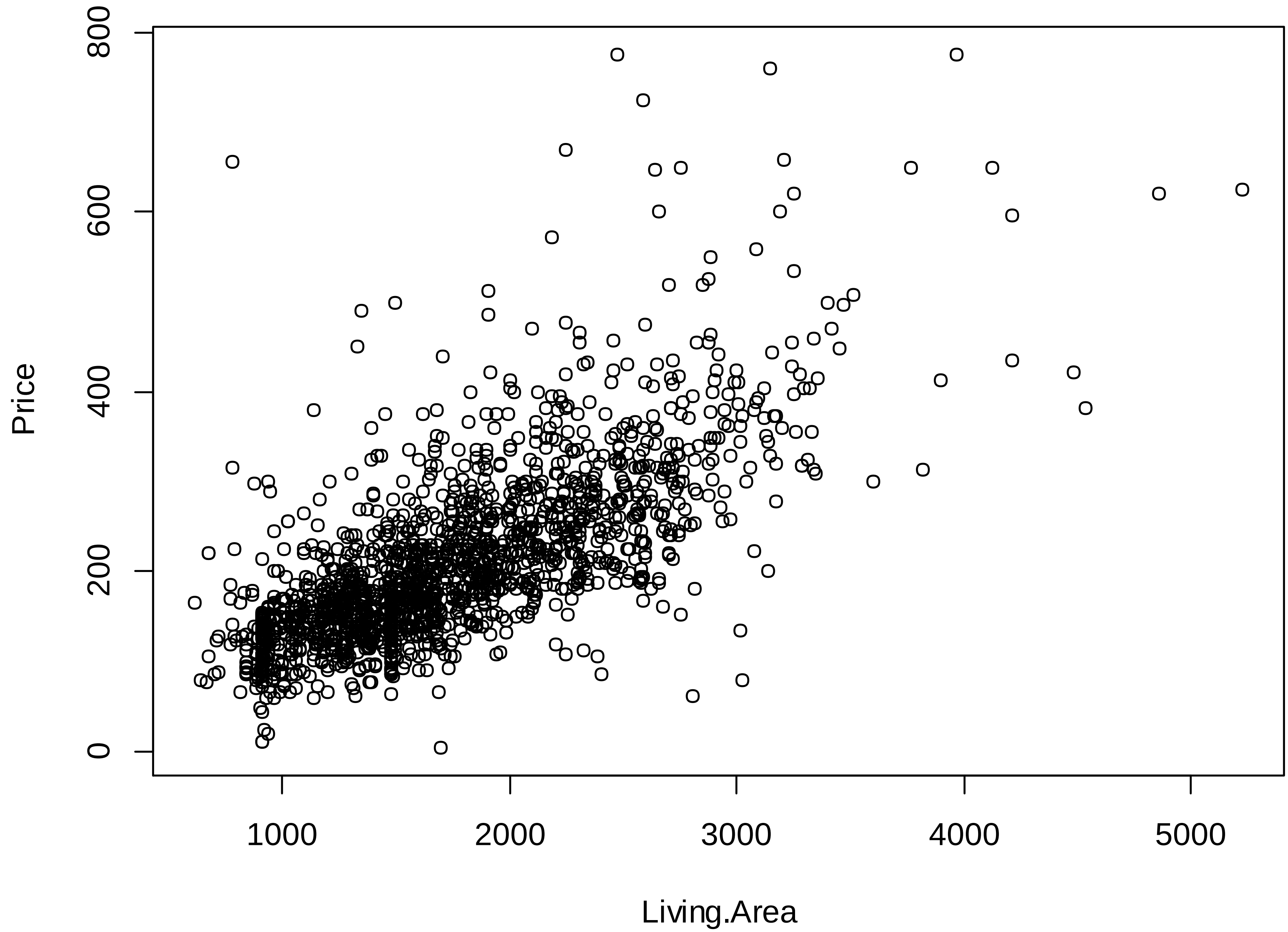
```
summary(lm(Price ~ Fireplaces, data = house))
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  171.824      3.234   53.13   <2e-16 ***
Fireplaces    66.699      3.947   16.90   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 91.21 on 1726 degrees of freedom
Multiple R-squared: 0.142,      Adjusted R-squared: 0.1415
F-statistic: 285.6 on 1 and 1726 DF,  p-value: < 2.2e-16
```

Questions:

- What is the marginal effect of *Fireplaces* on *Price*?
- How much does it cost to install a fireplace?
- Should I install a fireplace in my home?
- What the ? is going on here?
- What do you think the main determinant of *Price* should be?


Price
0
200
400
600
800
1000
2000
3000
4000
5000
Living.Area

The above plot was generated using the code:

```
plot(house$Living.Area, house$Price)
```

Is there a positive relationship between *Living.Area* and *Price*?

Now, estimate the model:

$$Price_i = \beta_0 + \beta_1 Living.Area_i + \epsilon_i$$

```
summary(lm(Price ~ Living.Area))


Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 13.439394   4.992353   2.692  0.00717 **
Living.Area  0.113123   0.002682  42.173  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 69.1 on 1726 degrees of freedom
Multiple R-squared: 0.5075,     Adjusted R-squared: 0.5072
F-statistic:  1779 on 1 and 1726 DF,  p-value: < 2.2e-16
```

- What is the marginal effect?
- Note the R^2 between the two regressions.
- What might be a problem with determining these two marginal effects separately?

```
cor(Living.Area, Fireplaces)
[1] 0.4737878
```

- If the variable *Living.Area* is excluded from the original regression, then it goes into the error term, u_i.
- If *Living.Area* and *Fireplaces* are positively correlated, then more fireplaces are just indicating a bigger house!
- That is, the error term is correlated with the "X" variable, and L.S.A. #1 is violated! The OLS estimator for β_1 in the first regression will be <u>biased</u>.

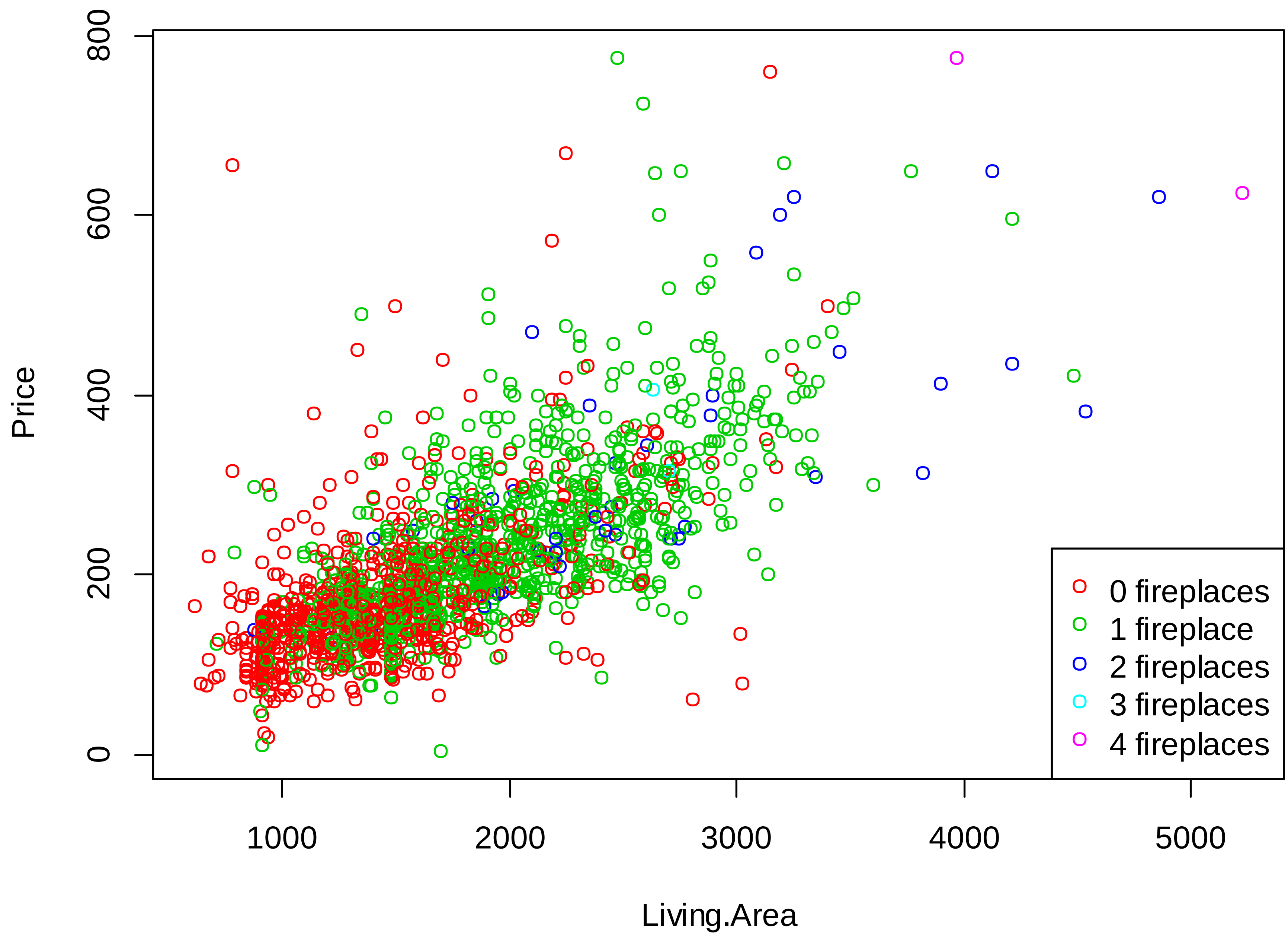
How can we take care of this problem? Include both variables in the model!

$$Price_i = \beta_0 + \beta_1 Fireplaces_i + \beta_2 Living.Area_i + \epsilon_i$$

```
summary(lm(Price ~ Fireplaces + Living.Area, data = house))
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 14.730146   5.007563   2.942  0.00331 **
Fireplaces   8.962440   3.389656   2.644  0.00827 **
Living.Area  0.109313   0.003041  35.951  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 68.98 on 1725 degrees of freedom
Multiple R-squared: 0.5095,     Adjusted R-squared: 0.5089
F-statistic: 895.9 on 2 and 1725 DF,  p-value: < 2.2e-16
```

- Notice how the estimated marginal effects have changed.
- Notice that *Fireplaces* is now a lot less significant.
- This is an example of <u>omitted variable bias</u> (OVB).


Price
Living.Area
0
200
400
600
800
1000
2000
3000
4000
5000
0 fireplaces
1 fireplace
2 fireplaces
3 fireplaces
4 fireplaces

Omitted Variable Bias

$$\hat{Price} = 171.82 + 66.70 \times Fireplaces, \ R^2 = 0.142$$
$$\quad (3.23) \quad (3.95)$$

$$\hat{Price} = 14.73 + 8.96 \times Fireplaces + 0.11 \times Living.Area, \ R^2 = 0.511$$
$$\quad (5.01) \quad (3.39) \qquad\qquad (0.003)$$

Several results have changed with the addition of the `Living.Area` variable:

- The estimated value of an additional fireplace has dropped from \$66,699 to \$8,962.
- The R^2 has increased from 0.142 to 0.5095.
- The estimated intercept has changed by a lot (but this is unimportant).
- There is a new estimated β: $b_2 = 0.11$. This means that, it is estimated that an additional square-foot of house size increases price by \$110.

Omitted Variable Bias

- Omitted variable bias (OVB) occurs when one or more of the variables in the random error term ϵ are related to one or more of the X variables
- A.5: X and ϵ are independent. OVB is a violation of this assumption, resulting in bias and inconsistency of OLS

- Suppose that X and Z both cause Y
- Suppose X and Z are correlated
- What happens when X changes?
- What is the problem with attributing changes in X to changes in Y?

Solution: include the omitted variable if possible