



### 5.3 – Dummy Variables

#### Dummy variable

–K and 1,000,000

- Takes on one of two values (usually 0 or 1)
- Dichotomous variable, binary variable, categorical variable, factor

$$D_i = \begin{cases} 0, & \text{if individual } i \text{ belongs to group } A \\ 1, & \text{if individual } i \text{ belongs to group } B \end{cases}$$

- Examples: gender, education, treatment, domestic, employed, insured, etc.
- Handwritten notes:* 0 if didn't complete HS, 1 if they did; 0 for domestic, 1 for foreign; control, treatment

In this section, we consider that the "X" variable is a dummy.

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#### A population model with a dummy variable

$$Y_i = \beta_0 + \beta_1 D_i + \epsilon_i \quad (5.13)$$

*Handwritten notes:* 0 or 1, true unobservable, slope?

- What is the interpretation of  $\beta_1$  here?
- Take a derivative? No, D isn't continuous
- What about  $\beta_0$ ? Still intercept
- Use conditional expectations

$$E[Y_i | D_i = 1] - E[Y_i | D_i = 0] = \beta_1 \quad (5.16)$$

*Handwritten notes:* Mean, mean

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#### Model w/ dummy

$$y = \beta_0 + \beta_1 D + \epsilon$$

$$E[y | D=0] = \beta_0 + \beta_1(0) = \beta_0 \rightarrow \text{mean for group } D=0$$

$$E[y | D=1] = \beta_0 + \beta_1(1) = \beta_0 + \beta_1 \rightarrow \text{mean for group } D=1$$

$$(\beta_0 + \beta_1) - \beta_0 = \beta_1$$

*Handwritten notes:* expected value, mean

↳ diff. in mean btw. two groups.

#### An estimated model with a dummy variable

Use OLS as before.

$$Y_i = b_0 + b_1 D_i + \epsilon_i \quad (5.17)$$

- $b_0$  is the sample mean ( $\bar{Y}$ ) for  $D_i = 0$
- $b_0 + b_1$  is the sample mean for  $D_i = 1$
- $b_1$  is the difference in sample means (be careful of the sign)

This means that, instead of using OLS, we could just divide the sample into two parts (using  $D_i$ ), and calculate two sample averages! So why should we use OLS? At this stage, it looks like we are making things more complicated than they need to be. However, in the next chapter we will add more X variables, so that we will not be able to get the same results by dividing the sample into two.

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#### Example: Gender wage gap using CPS

The current population survey (CPS) is a monthly detailed survey conducted in the United States. It contains information on many labour market and demographic characteristics. In this section, we will use a subset of data from the 1985 CPS, to estimate the differences in wages between men and women.

You will see many variables in the dataset. For now, we look at only a few:

- *dependent* wage - hourly wage
- *education* - number of years of education
- *gender* - dummy variable for gender

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difference in true mean ( $\mu_{D=1} - \mu_{D=0}$ )

$$\text{true model: } y = \beta_0 + \beta_1 D + \epsilon$$

$$\text{estimated: } y = b_0 + b_1 D + \epsilon$$

diff. in sample means  $\bar{y}_{D=1} - \bar{y}_{D=0}$

$$b_1 = \frac{\sum (y_i - \bar{y})(d_i - \bar{d})}{\sum (d_i - \bar{d})^2} = \bar{y}_{D=1} - \bar{y}_{D=0}$$



Load the data:

```
cps <- read.csv("https://rtdgodwin.com/data/cps1985.csv")
```

To run an OLS regression of wage on gender, use the following command:

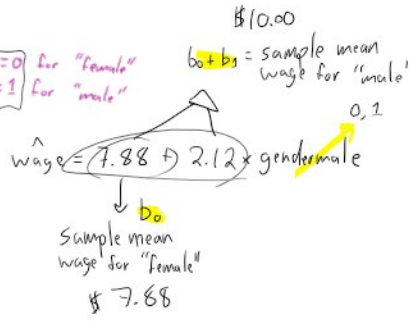
```
wage = beta_0 + beta_1 * gender + epsilon (pop. model)
```

```
summary(lm(wage ~ gender, data = cps))
```

```
Coefficients:
(Intercept) 7.8785 0.3218 24.55 < 2e-16 ***
gendermale  -2.1123 0.4372  -4.84 1.2e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.034 on 632 degrees of freedom
Multiple R-squared:  0.04219, Adjusted R-squared:  0.04038
F-statistic: 23.63 on 1 and 632 DF, p-value: 2.732e-06
```

Test: if there is a gender-wage gap, is there a diff. in wages btw. men and women?  
 $H_0: \beta_1 = 0 \rightarrow$  reject  $p\text{-val} < .001$



Defined: genderfemale = 0 if "male" = 1 if "female"

$b_0 = ?$  \$10.00  
 $b_1 = ?$  \$-2.12

From this output, you should be able to answer the following questions:

- What is the sample mean wage for males and for females?
- What is the interpretation of  $b_1$ ?

In class exercise: Test the hypothesis that there is no difference in the earnings of men and women.

We stated earlier that the results we obtain from regressing on a dummy variable are equivalent to what we would obtain by dividing the sample into two parts (by gender). Let's verify this using the CPS data. In R, create subsets for men and women:

```
cps_m <- subset(cps, gender == "male")
cps_f <- subset(cps, gender == "female")
```

Then take the difference in the sample mean wage between men and women:

```
mean(cps_m$wage) - mean(cps_f$wage)
[1] 2.116054
```

The difference is equal to  $b_1$ , which is 2.116! Also, note that the sample mean wage for women is  $b_0$ .

Sample mean wage for women is  $b_0$ :

```
mean(cps_f$wage)
[1] 7.878507
```

and the sample mean wage for men is  $b_0 + b_1$ :

```
mean(cps_m$wage)
[1] 9.994561
```

Exercise: A researcher defines the dummy variable in the opposite way. What are the new values for  $b_0$  and  $b_1$ ?

Handwritten answer:  $b_0 = 10.00$ ,  $b_1 = -2.12$

1  
le''

## 5.4 Reporting regression results

$$\hat{wage} = \underset{\substack{b_0 \\ (0.32)}}{7.88} + \underset{\substack{b_1 \\ (0.44)}}{2.12} \times \text{gendermale}, \quad R^2 = 0.042$$

$\swarrow$   $\searrow$   
s.e.( $b_1$ ) = s.e.( $b_1$ )

$$\frac{2.12}{0.44} = t\text{-stat for } H_0: \beta_1 = 0$$

This equation contains:

- Estimated  $\beta$ s
- Estimated standard errors
- $R^2$
- Everything you need to do a hypothesis test
- Example: test the hypothesis that there is no wage-gender gap

